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ARITHMETIC AND MENTAL DISCIPLINE

The impression is strong in many quarters—aggressively so in a few—that arithmetic has little or no value as mental discipline, in comparison with algebra, or geometry. There are those who concede willingly that the last two subjects have a value independent of their practical applications while they deny to arithmetic any such merit. It should be sufficient refutation of such view to show that the principles and applications of arithmetic are susceptible of development in substantially the same manner in which geometry is developed. We quote from a few well-known writers and teachers:

David Eugene Smith: "Every one is conscious that he got something out of the study (of arithmetic) aside from training in calculation and business application, that has made him stronger, and the few really scientific investigations that have been made as to the effect of mathematical study bear out this intuitive feeling."

J. W. A. Young: "What has been said as to the purpose and value of mathematics in general applies, properly interpreted, with as much force to arithmetic as to the other branches of mathematics."

A. Duncan Yocum: "In the first place, there appear to be no habits in algebra and geometry, generally useful in non-mathematical ways, that are not to be found in arithmetic as well. The habit of exact thinking, of successive judgments or conclusions, of analyses with a view to the identification of a familiar term, of instantly calling to mind the combination, or series of combinations, the identified term should suggest—the habit of perseverance in the face of difficulty or complexity, of looking for varied applications of a principle, of seeking the reason for each process or judgment—all these are not only common to the three subjects, but so fundamental for each as to constitute the chief equipment for specialization."

E. C. Brooks: "Then came the method of arithmetical analysis in that little gem of a book by Warren Colburn. It touched the subject as with the wand of an enchantress, and it began to glow with interest and

beauty. What before was full of routine, now became animated with the spirit of logic, and arithmetic was able to take its place beside its sister branch, geometry, in dignity as a science and value as an educational agency."

E. A. Lyman: "The author believes that mental training is an important feature in the study of arithmetic, but that the study need lose none of this training by the introduction of practical business methods."

In claiming that arithmetic may be so taught that its yield in mental discipline need not be inferior to that produced by algebra and geometry we are not concerned with the question, in what grade, or grades, such teaching should be done. That is a question of school administration. We are only concerned with the fact that the material of ordinary arithmetic lends itself to the same types of mental procedure on the part of the student as those through which he is usually carried by his work in geometry or algebra.

Perhaps many hold that arithmetic is ineffective as a mental trainer because it is difficult to separate practical aims from mechanical methods. When once we assume that the practical application of a study is of paramount importance to everything else about it, at once a multitude of influences begin operating to mechanize the study.

Every rule of arithmetic may be made to appear as a link in a logical process based in a body of assumptions. If power is acquired by the mind from its repeated construction of logical highways between hypotheses and conclusions related to lines and angles and polygons, it is unreasonable to deny that equivalent power is attainable by similar constructions between hypotheses and conclusions related to numbers and defined number relations.—S. T. S.

THE JOY OF MATHEMATICS

By JAMES McGIFFERT

Six hundred and seventy-six years ago, Roger Bacon, the British Philosopher, wrote these words: "Divine Mathematics, which alone can purge the intellect and prepare the mind for the reception of all knowledge." True words these, although written at a time when scientific knowledge was only in its infancy.

What is the Joy of Mathematics: It is its victory, it is the realization that it has come to its own.

Time was when mathematics, beyond the four fundamentals of arithmetic, and the so-called Rule of Three, was relegated to the cloister, the monastery, and the attic, where deluded enthusiasts wasted their time, their minds, their souls and their bodies in solving abstruse mathematical problems, just for the love of the work.

Time was when the old quatrain was considered quite sensible:

"Multiplication is vexation,
Division's twice as bad;
The Rule of Three perplexes me,
And Fractions drive me mad."

How gloriously all has changed, until now mathematics has come to occupy a place preeminent among the sciences. Let us review some of the developments which have led to such great joy in the hearts and minds of mathematicians.

In 1614 Baron John Napier, Laird of Merchiston, Edinburgh, published his wonderful invention of logarithms, whcih did more to revolutionize and simplify numerical calculations than any other discovery in pure science.

Professor Henry Briggs, of Oxford, suggested an improvement on the work of Napier, by adapting logarithms to the base 10, and thus made the work much more easily applicable to numerical computations. Professor Briggs rode on horseback from Oxford to Edinburgh, in the summer of 1614, and coming into the presence of the great Scotch mathematician, expressed himself as follows: "Lord Napier, I have never seen anything which pleased me more, or made me more wonder than your remarkable invention of logarithms, and I should love to suggest that you adapt them to the base 10." Here was demonstrated a spirit on the part of Napier, which has not been evinced by many

scientists, or men of any walks of life. Instead of being jealous at Professor Briggs' suggestion, Napier said that he would be happy to accept the suggestion, and would be very glad to have him cooperate in the work, which was prodigious. Thus was cemented a friendship, which was productive of wonderful results. Napier and Briggs worked out their tables, and from that time on we have been given tables of logarithms to as many as 40 or more places of decimals, which have simplified the calculations of the astronomer and physicist most tremendously. What joy must have come to the heart of Napier, as he saw this new idea take root and become the greatest help to all people.

Later on, the great Sir Isaac Newton lived and worked and dreamed. But although he did bring forth the greatest discovery of all time, namely, the law of universal gravitation, his joy was marred by many arguments and discussions of an antagonistic nature, in so much that Newton wrote these words, concerning the study of Philosophy: "Philosophy is such an impertinently litigious lady, that a man had as well be engaged to law suits as to have to do with her." Thus was his joy dimmed. His invention of the Infinitesimal Calculus was also held back for many years. Before his work which he called Fluxions, was published, the continental mathematician, Leibnitz, produced his work on the Calculus. From that time for one hundred and thirty years the conflict between the followers of Newton, and of Leibnitz raged, until three young Cambridge graduates, Herschell, Babbage and Peacock, made a translation into English of the Calculus of LaCroix, and thus gave the English speaking people the work of Leibnitz.

But through all the controversies and arguments a glow of joy lighted up the soul of Newton, who wrote that he felt like a child playing on the seashore, picking up pebbles, and that he was delighted at the discovery of a smoother or more colorful pebble, whereas the whole ocean rolled on before him, with its limitless depths of wonder.

After Sir James Jeans, in his Mysterious Universe, has traversed the fartherest reaches of space, and has passed beyond the fixed stars, and the most remote nebular cluster, and has come back to study the intimate rescesses of the atom, he concludes most reverently that the Maker of this universe, the Creator of all this harmony of wonder must be a great Pure mathematician.

Sir Arthur Eddington, Great Britain's greatest astro-physicist, read a paper on the Interior of the Stars, before the International Mathematical Congress in Toronto, in 1924. His conclusion was most startling. It was that the density of the companion star of Sirius was about 60,000

times that of water. This means that it weighs nearly 3000 times as much, volume for volume, as the heaviest known element on earth. Such an amazing conclusion needed confirmation, and Sir Arthur Eddington suggested that it be attempted to prove its accuracy by the acid test of Einstein's theory that the lines in the spectrum are shifted toward the red end, by the gravitation of a heavy body. The experiment was difficult to perform, but Dr. Adams, of Mount Wilson, succeeded in obtaining the result, and his conclusion was a perfect corroboration of the calculations of Sir Arthur. What joy must have come to the heart of such a seeker after truth, when such confirmation came to him.

Such joy cannot be purchased with money. It is soul joy, not of the earth, earthy.

During the past century three great names have come to be very well known. They are Fourier, Legendre, and Bessel.

Fourier developed his trigonometric series, which at once had its perfect application in the solution of problems in the flow of heat, or of electricity. The mathematical work of Fourier has now come to be most essential in all telephonic and radio work.

Legendre found a solution of so-called zonal harmonic problems and the mathematical investigations he carried out have made him famous. At once his work found application in the solution of all problems in electric potential in space.

Bessel developed his cylindrical harmonics, and his researches have done much to aid the physicist and engineer in solving all sorts of problems involving cylindrical questions.

It was wonderfully fortunate that practically the work of these three great men found such large application, but the joy of accomplishment, the happiness resulting from success, that was the most perfect and satisfactory reward such work ever can have. Nothing is so pleasing as the solving of a knotty problem. We all know this. Let us all believe that mathematicians, more than any others, have had joy in solutions of the problems, which have been most useful in all branches of applied science.

Seeing then that you are compassed about with so great a cloud of witnesses to the joy of mathematics, will not you who teach our young students, endeavor to fill your class rooms with an atmosphere of joy? Of course most of you are overtaxed with what you consider drudgery, but if you can come to realize that you are building the mathematical habit into the minds of our pupils, you will come to have deep joy in your own hearts.

Try to make your problems interesting to boys and girls. Do not bother about problems which ask how long it will take C to do a piece of work, after A, who can do it alone in 6 days, and B in eight days have fallen by the wayside, but give problems concerning the strike-outs of Carl Hubbell, the home runs of Babe Ruth, the end-runs of the favorite football players of the present season, or the basketball games played by your girls. Any problems which will make the abstract concrete will help marvelously.

When mentioning the prediction of Einstein that light rays are bent toward a heavily gravitating body, such as the sun, recite the fine parody on Twinkle, twinkle," namely:

Twinkle, twinkle, little star,
How I wonder where you are.
High above I see you shine,
But according to Einstein,
You are not where you pretend,
You are just around the bend.
And your sweet, seductive ray
Has been leading men astray
All these years. Little Star,
Don't you know how bad you are?

When you are discoursing on the glories of the heavens, read them this story of Astronomy Made Easy:

If you cannot tell a planet from the Bird that's called a ganet,
Or distinguish Sagittarius from Mars;
If the birds in that strange Zoo, all look alike to you,
And you dub the whole caboodle just as stars;
If you cannot find the Lion, or correctly trace Orion,
Or discern the jewelled belt he proudly wears,
Or the Big and Little Hounds in their happy hunting grounds,
Nightly chasing up the Big and Little Bears;
If you cannot tell the Dipper from your Grand dad's old felt slipper
And to name the Constellations you would fail,
There is one thing you can do, and be mighty sure it's true,
You can always tell a comet by its tail.
Its airy, fairy, winking, blinking, flowing, glowing tail,
Its wiry, fiery, gleaming streaming flaring glaring tail.

When a wee pupil is sad and weepy just pat him or her on the back and quote, "Weeping may endure for a night, but joy cometh in the morning."

So permit yourself to be impregnated with the Joy of Mathematics, that it will shine out in all your actions. Then will you believe and live the following application of the four fundamentals of mathematics.

Plus Contentment,
Minus Worry,
Multiplied into Service,
Divided by Liberality.

This should be the formula for a man.

—o—

A HISTORY OF THE DEVELOPMENT OF MATHEMATICS IN THE FIELD OF ECONOMICS

By IDA BELL SHAW
Ruston, La.

(Presented as a Master's Thesis in the Mathematics Department of Louisiana State University.)

Introduction

"It is clear that Economics, if it is to be a Science at all, must be a mathematical science."[1] Gossen insists that the mathematical treatment is the only sound one, and must be applied throughout.

"The systematization which occurs in a theoretical science, as we may properly call it in order to distinguish it from a natural or an applied science, is a process which is apt to come late in the develop ment of a subject. Evidently some fields of knowledge are hardly ready for it, for it is typified by a free spirit of making hypotheses and definitions rather than a mere recognition of facts. But when we find this feeling for hypothesis and definition and, in addition, become involved in chains of deductive reasoning, we are driven to a characteristic method of construction and analysis which we may call the mathematical method.

"It is not a question as to whether mathematics is desirable or not in such a subject. We are in fact forced to adopt the mathematical method as a condition of further progress."[2]

[1]William Stanley Jevons, "Mathematical Character of the Science," *The Theory of Political Economy*, p. 3.
[2]Griffith E. Evans, "General Concepts and Methods," *Mathematical Introduction to Economics*, p. 113.

With application of mathematics, the economists have succeeded in substituting an attractive and ingenious method for the somewhat halting hypothesis put forth by the classical theorists. Mathematical political economy, "not content with seeking relations of mutual dependence between isolated facts, claims to be able to embrace the whole field within its comprehensive formula."[3]

The interdependence of economic facts necessitates mathematical treatment. Professor Henry L. Moore brought out this fact clearly in his statement: "A comprehensive treatment of economic questions in a changing society must take cognizance of the interdependence of all types of economic change, and the only kind of treatment that will lead to rational forecasting and control is mathematical in character."[4]

The introduction of diagrams and algebraic formulae into the works of the economists is the most notable change in economic science in recent years. The mathematical school does not make much use of numbers. It confines itself to algebraical notation and geometrical figures; that is, to the consideration of abstract quantities. To write down a problem in the form of a mathematical equation is to show that a problem can be solved and give conditions under which solution is alone possible. Beyond this the economist never goes.

"The mathematical method has marked advantages in the definition and analysis of concepts, in the discovery and presentation of a general view of the economic field, and in the discovery of appropriate quantitative methods. It may be affirmed that the pure mathematical theory of economics stands in the same relation to the literary theories as the Sperry gyroscopic compass does to the ordinary variety of Mariners' compasses."[5]

Jevons contends "that all economic writers must be mathematical so far as they are scientific at all, because they treat of economic quantities, and the relations of such quantities, and all quantities and relations of quantities come within the scope of mathematical character of treatment, continually betray in their language the quantitative character of their reasonings. What, for instance, can be more clearly mathematical in matter than the following quotation from Cavine's chief work: 'We can have no difficulty in seeing how cost in its principal elements is to be computed. In the case of labour, the cost of pro-

[3]Gide and Bist, "The Mathematical School," *History of Economic Doctrines*, 0. 533.
[4]Henry Ludwell Moore, *Synthetic Economics*. See Introduction, p. 2.
[5]Henry Shultz, "Mathematical Economics and the Quantitative Method," *Journal of Political Economy*, Vol. 35, p. 703.

ducing a given commodity will represent by the number of average labourers employed in its production."[6]

He also says "that our science must be mathematical, simply because it deals with quantities. Wherever the things treated are capable of being greater or less, there the laws and relations must be mathematical in nature. The ordinary laws of supply and demand treat entirely of quantities of commodities, demanded or supplied, and express the manner in which the quantities vary in connection with the price. In consequence of this fact the laws are mathematical. Economists cannot alter their nature by denying them the name; they might as well try to alter a red light by calling it blue. Whether the mathematical laws of Economics are stated in words, or in usual symbols, x, y, z, p, q, etc., is an accident, or a matter of mere convenience."[7]

"The symbols of mathematical books are not different in nature from language; they form a perfected system of language adapted to the notions and relations which we needed to express. They do not constitute the mode of reasoning they embody; they merely facilitate its exhibition and comprehension. If, then, in Economics, we have to deal with quantities and complicated relations of quantities, we must reason mathematically; we do not render the science less mathematical by avoiding the symbols of algebra—we must refuse to employ, in a very imperfect science, much needing every kind of assistance, that apparatus of appropriate signs which is found indispensable in their sciences."[8]

Price says, "By emphasizing its limiting conditions, the historical treatment has checked the misapplication of theory; and the mathematical treatment proceeding from a different starting point by a different road has reached the same goal, and tended to induce greater precision of statement."[9]

Keynes says: "It may indeed be added as amongst the special advantages of mathemtical methods that they lead not only to accuracy and precision, but also to conciseness and the avoidance of circumlocution."[10]

It is difficult for any one at the present date to realize what was the state of economic science in the days before Jevons and Walras familiarized students with the use of exact reasoning in Economic

[6]William Stanley Jevons, *op. cit.*, Preface, p. xxi.
[7]*Ibid.*, p. 4.
[8]*Ibid.*, p. 5.
[9]L. L. Price, *Economic Science and Practice*, p. 309.
[10]John Neville Keynes, "Mathematical Methods," *Scope and Method of Political Economy*, p. 260.

Science, expressed in the accurate language of mathematics. Errors in pure economics are due either to incorrect assumptions or to invalid deductions. The use of mathematical symbols enables the reader to see clearly what the underlying assumptions are, and to follow easily the logical deductions from those assumptions. The use of mathematics is the best means of getting rid of both inaccuracy of assumption and looseness of deduction.

CHAPTER I

History of Political Economy

In the ancient world we have only slight sketches of such a science as Economy. In the Middle Ages no advance was made. The idea itself was lost. At the end of the fourteenth century, when the medieval order was breaking up, and the modern industrial system was beginning, a French bishop revived the teachings of the great master of ancient speculation—Aristotle—but with no immediate success. Early in the sixteenth century, the famous astronomer and mathematician, Copernicus, wrote a treatise on the coining of money, again based on the principles of Aristotle. Toward the end of the century modern economic science began to take shape in the great mercantile theory which prevailed more or less until the days of Adam Smith. In the seventeenth century, when medieval conditions were rapidly giving place to modern conditions, attention began to be directed toward economic problems. The theory of commerce grows in this period, but this theory found an opponent in Sir William Petty, by far the most important figure of political economy in the seventeenth century. By the middle of the eighteenth century, the extreme mercantile theory had well nigh succumbed.

In the latter part of the eighteenth century we have the beginning of our important economic works. Adam Smith is the outstanding economist of this time. He is given credit for laying the foundations of the science of political economy. Connected with the name of Adam Smith are the names of Thomas Malthus and David Ricardo. These three constitute the trinity of classical economists. After these three came John Stuart Mill, who filled in details of the previous writers and expounded this branch of knowledge.

The work in the field of economics now takes quite a turn in trend. The work of the Mathematico-Economists claims the attention of the economic public. The outstanding writers through the nine-

teenth century are Antoine Augustin Cournot, Hermann Heinrich Gossen, William Stanley Jevons, and Marie Esprit Leon Walrus.

Today the mathematical method can claim representatives in in every country. Marshall and Edgeworth in England; Laundhardt, Auspitz, and Lieben in Germany; Pareto and Barone in Italy; Irvin Fisher, Henry L. Moore, Griffith C. Evans, C. F. Rces, and Henry Schultz in the United States; and Bortkevitch in Russia. France, however, the country of Cournot and Walras, has no mathemtical economists, unless we mention Auspetit, whose work *Theorie de la Monnaie*, although dealing with a special subject, contains a general introduction.

It is the Mathematico-Economists that we are going to deal with in this study.

CHAPTER II

Traces of Mathematics in the Writings of the Classical Economists

Before we go into the works of the men of the real mathematical school of economics, let us first take notice of traces of mathematical thought in the writings of the classical school.

Adam Smith in the fifth chapter of the First Book of *Wealth of the Nations* continually argues about "quantities of labour," "measures of value," "measures of hardships," "proportion and equality," etc. All of the ideas are mathematical. In the Second Book of *Wealth of the Nations*, we read: "The produce of land, mines, and fisheries, when their natural fertility is equal, is in proportion to the extent and proper application of the capitals employed about them. When the capitals are equal, and equally well applied, it is in proportion to their natural fertility."[1] Every use of the word "equal" or "equality" implies the existence of a mathematical equation. The equation is simply an equality. Every use of the word "proportion" implies a ratio expressible in the form of an equation.

Ricardo, in the thirteenth chapter of his *Political Economy*, states, "It is true that the least productive mines, those which paid no rent, could no longer be worked, as they could not afford the general rate of profits till the relative value of gold rose, by a sum equal to the tax."[2]

[1]Adam Smith, *Second Book of the Wealth of the Nations*, p. 16.
[2]David Ricardo, *Political Economy*, p. 172.

Mathus in his book on *Political Economy* says, "Taking an average of the five preceding years in which prices of wheat are noticed, including 1575, having previously averaged the several prices in the same year, as before, it appears that the price of the quarter of wheat was I 1. 2s. 2d., which is 25.9½d. the bushel, and 8¼d. the peck. At this price, a day's labour would purchase a peck of corn within a farthing, or 16/17 of a peck."[3]

All three of the famous economists have used mathemtical language, but yet on examining copies of their works, I have failed to find a single mathematical symbol, formula, or equation. Yet these three, by the use of mathematical modes of thought, give an explicit recognition of the mathematical character of the science of economy—almost a necessary condition of any real improvement of the theory of the science.

Many readers may be surprised to know that John Stuart Mill used mathematical symbols, but in his book on *Principles of Political Economy*, Book III, Chapter xviii, the difficult and tedious chapter in which he leads the reader through the "Theory of International Values" by means of yards of linen and cloth, the reader will find that Mill at least yields to mathematics, and expresses himself concisely and clearly by means of equations between m, n, p, and q. His mathematics is very crude; yet there is some approach to a correct mathematical treatment, with the result that this chapter, however tedious and difficult, will probably be found the truest and most enduring part of the whole treatise.

Mill says, "The whole of the commodities which the two countries (i. e. England and Germany) can respectively make for exportation, with the labor and capital thrown out of employment by importation will exchange against one another.

This law and the three different possibilities arising from it in respect to the division of the advantage, may be conveniently generalized by means of algebraical symbols, as follows:

> Let the quantity of cloth which England can make with the labor and capital withdrawn from the production of linen be n.
>
> Let the cloth previously required by Germany (at the German cost of production) be m.
>
> Then n of cloth will always exchange for exactly 2m of linen
>
> Consequently, if n equals m, the whole advantage will be on the side of England.

[3]T. R. Mathus, *Political Economy*, p. 209.

If n equals 2m, the whole advantage will be on the side of Germany.

If n be greater than m, but less than 2m, the two countries will share the advantage; England getting 2m of linen where she before got only n; Germany getting n of cloth where she before got only m."[4]

It is almost superfluous to observe that the figure 2 stands where it does, only because it is the figure which expresses the advantage of Germany over England in linen as estimated in cloth, and (what is the same thing) of England over Germany in cloth as estimated in linen. If we had supposed that in Germany, before the trade, 100 of cloth exchanged for 1,000 instead of 200 linen, then n (after the trade commenced) would have exchanged for 10 m instead of 2m. If instead of 1,000 or 200 we had supposed only 150, n would have exchanged for only 3/2m. If (in five) the cost value of cloth (as estimated in linen) in Germany, exceeds the cost value similarly estimated in England, in the ratio of p to q, then will n after the opening of the trade, exchange for p/qm.[5]

Here Mill has employed undisguised mathematical formulae.

The quantitative, and therefore in a broad sense mathematical character of economic reasonings might be illustrated by Mill in his treatment of supply and demand as regulators of value. He insists, for example, that the idea of a ratio between supply and demand is out of place, that what is really involved is an equation.

In the writings of Canard and Whewell, two of the lesser important economists, "we find plenty of symbols and equations with no result of value, owing to the fact that they simply translated into symbols the doctrines obtained, and erroneously obtained, without their use.[6] So in the case of these two writers they misunderstood and inverted altogether the function of mathematical symbols.

(Continued in NEWS LETTER No. 3, this volume.)

[4]John Stuart Mill, *Political Economy*, p. 118.
[5]*Ibid.*, p. 119.
[6]William Stanley Jevons, *Political Economy*, Preface.

AN EVALUATION OF A CERTAIN DOUBLE INTEGRAL

By RAYMOND GARVER

In the *American Mathematical Monthly*, in 1914, the following note appeared in the department of Questions and Discussions:

"In the process of solving a certain physical problem Professor H. S. Uhler, of Yale University, was led to the definite integral

$$(1) \qquad \int_0^a \int_{a-x}^{a+x} \frac{e^{-cy}}{y} (a^2 - x^2)\, x\, dy\, dx,$$

for which he found the value

$$(2) \qquad \frac{1}{c^2}\left| a^2 - \frac{3a}{c} + \frac{1}{c}\left(a + \frac{3}{2c}\right)\left(1 - c\sqrt{-ac}\right) \right|,$$

a and c being positive constants. Professor Uhler would like to see how other persons attack the problem of evaluating the intergal".

Two solutions were published, one by Professor A. M. Harding[1], and one by Professor Uhler[2]. Professor Harding wrote the integral as

$$(3) \qquad \int_0^a (a^2 - x^2)\, x\, f(x) dx,$$

where f(x) is obviously an abbreviation for

$$(4) \qquad \int_{a-x}^{a+x} \frac{e^{-cy}}{y}\, dy,$$

and integrated by parts. I shall not attempt to reproduce the details of the work here. Professor Uhler differentiated under the integral sign with respect to the parameter c.

It seems to me that a somewhat more instructive method of attack, at least with an integral calculus class in mind, is to interpret the integral (1) as a volume, and to evaluate it by an interchange of the order of integration based on a study of the fundamental area above which the volume lies. I might say, in fact, that the main point of this note is to stress the importance of problems of this type in the

[1]Vol. 21 (1914), 342-3.
[2]Vol. 22 (1915), 35.

integral calculus class. If a student can take an integral of the form $\int\int f(x,y)\,dy\,dx$, which can not be evaluated directly because of the form of the function f(x,y), and by interpreting it as a volume, obtain another set-up which leads to an evaluation, he shows pretty clearly his understanding of the representation of volumes as double, or iterated, integrals. Such problems may be used as a valuable supplement to the usual problems leading directly from volume to integral. Of the various calculus texts which I am familiar with, Love seems to be the only one to formulate problems of this type, though I have no doubt that there are others. Among Love's examples are

$$\int_0^a \int_x^a \frac{e^y}{y}\,dy\,dx, \quad \int_0^1 \int_x^1 e^{y^{-2}}\,dy\,dx, \quad \int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\,dy\,dx,$$

the last of which is to be transformed to polar coordinates, or perhaps we should say cylindrical coordinates if we are thinking of the problem as a three-dimensional problem.[1] A practical example is the (simplified) probability integral,

$$\int_0^\infty e^{-x^2}\,dx$$

a double integral is set up whose value is easily seen to be I^2, and this is evaluated by transforming to polar coordinates.[2] Of course, the infinite upper limit requires the introduction of a limiting step.

We return now to the discussion of (1). It may be thought of as representing the volume under the surface.

$$z = \frac{e^{-cy}}{y}(a^2-x^2)x \tag{5}$$

and above the area in the XY-plane which is bounded by the lines $y=a-x$, $y=a+x$, $x=a$. The surface is non-negative in this region and is single-valued except at (a,0). Strictly speaking, z is not defined at this point, but the reader can convince himself easily that, as (Px,y) approaches (a,0) from within the fundamental area, z approaches any desired value from 0 to $2a^2$. As P approaches (a,0) along $x=a$, z is always zero, and if the approach is along $y=a-x$, z approaches

[1] Love, Differential and Integral Calculus, 1918, p. 266-68.
[2] See, for example, Woods, Advanced Calculus, 1926, p. 153.

$2a^2$. We may, if we desire, think of the surface as containing the line segment joining (a, 0, 0) to (a, 0, $2a^2$). The integral $\int\int f(x,y)\, dy\, dx$ represents a volume bounded by a surface, a cylinder, and a fundamental area, and it is permissible, though not usual, for the surface to contain part or all, of an element of the cylinder.

If we wish to change the order of integration, we see that two integrals are required. The volume becomes, in fact,

$$(6) \qquad \int_0^a \int_{a-y}^{a} z\, dx\, dy + \int_a^{2a} \int_{y-a}^{a} z\, dx\, dy,$$

where z is given by (5). After the first integrations with respect to x a considerable simplification results, and the two integrals combine into one, namely

$$(7) \qquad \int_0^{2a} e^{-cy} \left(\frac{y^1}{4} - ay^2 + a^2y \right) dy.$$

The evaluation of (7) is perfectly straightforward, requiring three integrations by parts of the same type. If $\int e^{-cy} y^n dy$ is integrated a familiar reduction formula is obtained which may be used to advantage. The result is easily seen to be equivalent to (2), though it does not appear at first in quite this form.

THE SOUTHERN INTERCOLLEGIATE MATHEMATICS ASSOCIATION

By I. MAIZLISH
Centenary College

A new organization, known as the Southern Intercollegiate Mathematics Association, was organized at Centenary College, Shreveport, Louisiana, on October 21, 1933. This association embraces the colleges and universities in the States of Mississippi, Louisiana, Texas, and Arkansas. Its chief object is to promote and support contests in mathematics, and to encourage mutual and intellectual fellowship among its members.

Competition in Mathematics is not new. In earlier times individual mathematicians often engaged in such contests. Again, intercollegiate contests in general are not new. Among such contests, one may mention

intercollegiate debating contests, the contests in writing between the Universities of Maine, New Hampshire, and Vermont in 1926, and the contests in literature between Harvard and Yale in 1928. Although the writer was unaware of this until about two weeks ago, Professor Carl E. Seashore had outlined a plan for intercollegiate contests in *School and Society*, July, 1928. It is inevitable that intercollegiate contests of all kinds should have certain elements in common because, presumably, the objectives to be attained are either the same or similar. Nevertheless, what is perhaps new in education is the *organization* of this association.

Membership in the Association is restricted to universities and senior colleges in these States. The four States have been arbitrarily divided into four regions in such manner as to make the number of members in each region approximately the same. The officers of the Association have been so chosen as to make it thoroughly representative.

The contests have been divided into regional contests and final contests. The regional contests consist of preliminary examinations and the final contests of final examinations in each of the following subjects: college algebra, plane trigonometry, plane analytic geometry, calculus, and a comprehensive examination in all of these subjects. The preliminary examinations are held at the individual institutions at the same time, and are the same for all institutions.

The regional winners are eligible for the final contests which are held at the same time and place as the annual meeting of the Association. It was decided that, in both the preliminary and final examinations, the "grade" shall be the average grade obtained by a given institution in all five examinations. In both the preliminary and final examinations, the winners will be the *institutiton* and not the individual students.

The Association has two kinds of dues: (a) each institution pays annual dues of two dollars; and (b) the head of the Department of Mathematics of each institution pays annual dues of one dollar.

The contests are restricted to undergraduate students. The Constitution of the Association defines, among other things, the eligibility of students, the types of contests, prizes, and the methods of grading and making out examinations. The Association obligates each member to recognize the students taking part in these contests by awarding to each of them a "letter" or an equivalent insignia.

The interest manifested in this Association may best be seen from some quotations:

"Our civilization has a basis in mathematics. Accurate thinking is promoted by it; therefore, I would encourage any movement which has as its purpose the fostering of interest in the field."—President Pierce Cline, Centenary College.

"I heartily approve of the idea of an organization to promote interest in intercollegiate contests in mathematics. Such a thing should be a decided aid in increasing interest in mathematics."—Dean John A. Hardin, Centenary College.

"It goes without saying, that it would be very pleasing to me to see an increase in 'intellectual' contests among students. We need to give greater prominence to scholarship."—Chancellor Alfred Hume, University of Mississippi.

" I think your project is fine."—Dr. P. K. Smith, Louisiana Polytechnic Institute.

"It is a commendable movement"—Professor S. T. Sanders, Louisiana State University.

"I am very interested in your project."—Dean B. E. Mitchell, Millsap College.

"I think this is a very important movement."—Professor G. D. Estes, Arkansas State Teachers College.

"The idea of the Southern Intercollegiate Mathematics Association is an excellent one."—Professor Hosford, University of Arkansas.

"I heartily endorse the suggestion made and I am ready to help in any way possible to carry on the good work.—Professor Jennie Tate, McMurry College.

Intellectual competition does not meet with universal approval. Those who do not favor such competition have been somewhat too severe in their criticisms. Perhaps the most severe criticism intellectual contests have been subjected to is that "they seem predicated upon the conviction that popularity means success." I believe that no educator will claim that contests of this kind will deepen scholarship, any more than he will maintain that colleges and universities, in general, produce geniuses. The unsympathetic mind will find fault with such a movement, but on examination it will be found that our critics do not really enter into such a movement with the spirit with which such contests are intended to be executed, and which is presupposed on the part of every member and its representatives. Any one who examines the possibilities of such contests as are contemplated in the Southern Intercollegiate Mathematics Association with a sympathetic attitude

will readily see that, if *properly conducted,* among other things, such contests will ultimately

1. give more importance and greater dignity to scholastic attainments;
2. offer some basis for comparison among the various mathematics departments;
3. divert some of the interest, enthusiasm and energy from athletics, (especially football), to the intellectual domain;
4. increase, or tend to increase, interest in mathematics;
5. help spread the conviction that our enduring pleasures and interests are in the last analysis intellectual;
6. benefit the schools participating in these contests by bringing them into closer intellectual cooperation;
7. "better acquaint the public with and perhaps interest it in" the work in our higher institutions of learning; and, finally,
8. benefit the mathematics departments themselves in various ways, by being represented in these contests.

No one will seriously doubt that such contests will be an incentive to better scholarship in mathematics. There is good reason to believe that, on the assumption that these contests are conducted in the proper way, other benefits expected from them will also come.

Our attitudes toward what we do, what we are, and what we see—in short, our attitudes toward life and its concommitants—are the most important of all things; and the attitude we take toward such an association, and that with which we carry on its work will determine its success as well as what it may be expected to achieve.

CONSTITUTION OF THE SOUTHERN INTERCOLLEGIATE MATHEMATICS ASSOCIATION

Article I. Name

Sec. 1. This organization shall be known as the Southern Intercollegiate Mathematics Association.

Article II. Object

Sec. 1. The object of the Association shall be to promote and support contests in Mathematics and to encourage mutual and intellectual fellowship among its members.

Article III. Membership

Sec. 1. Any University or Senior College in Louisiana, Mississippi, Texas and Arkansas, which is recognized by some standardizing agency, shall be eligible for membership in this Association.

Sec. 2. To gain membership in this Association, it shall be necessary for the school applying to receive two-thirds of the votes cast by those present at the annual meeting, each member having one vote.

Article IV. Dues

Sec. 1. Each member of the Association shall pay annual dues of two dollars ($2.00), and each Head of the Mathematics Department shall pay annual dues of one dollar ($1.00).

Sec. 2. An institution will be disqualified for membership if both the institution and the Head of the Mathematics Department are in arrears for two years.

Article V. Regions

Sec. 1. The members of the Association shall be divided into four regions in such a manner as to make the number of members in the respective regions approximately the same.

Article VI. Officers

Sec. 1. The officers of the Association shall consist of a President, a Vice-President, a Secretary-Treasurer and an Executive Council consisting of one representative from each region into which the four states have been divided. The President, Vice-President, and Secretary-Treasurer shall be ex-officio members of the Executive Council.

Sec. 2. The officers shall be chosen in such manner and in accordance with such geographical distribution as to make the Association thoroughly representative and avoid localization.

Sec. 3. The duties of these officers shall be such as ordinarily devolve upon them.

Sec. 4. The President, Vice-President, and Secretary-Treasurer shall hold office for two years and be eligible for reelection.

Sec. 5. The members of the first Executive Council shall be elected for one, two, three, and four years, respectively, and thereafter for a term of four years, and shall be eligible for reelection.

Article VII. Power of Council

Sec. 1. The Executive Council shall have full authority to make adjustments of all complaints and difficulties that may arise. The members of the Association are honor-bound to abide by every decision rendered by the Council.

Article VIII. Meetings

Sec. 1. The Association shall hold an annual meeting at a place decided upon at the previous meeting of the Association.

Sec. 2. The time of the annual meeting shall be decided by the Executive Council.

Article IX. Contests

Sec. 1. There shall be a regional contest and a final contest each year.

Sec. 2. The regional contest shall consist of preliminary examinations and the

final contest of final examinations in each of the following subjects: (1) college algebra; (2) plane trigonometry; (3) plane analytic geometry; (4) calculus; and (5) a comprehensive examination in all of these subjects.

Article X. Grade

Sec. 1. In both the regional and final contests, the *"grade"* shall mean the average grade obtained by an institution in all five examinations given.

Article XI. Winners

Sec. 1. The institution in each region that makes the highest grade in the preliminary examinations shall be considered the regional winner.

Sec. 2. The regional winners shall be eligible for the final contest which is to be held at the same time and place as the annual meeting of the Association.

Sec. 3. The institution that makes the highest grade in the final contest shall be considered the winner of the contest.

Article XII. Examinations

Sec. 1. The preliminary examinations shall be held at the individual institutions and shall be the same for all institutions. These examinations must be given within two days of the time specified by the Executive Council.

Sec. 2. An institution must partake in all of the preliminary examinations to be eligible for the final contest.

Sec. 3. The examinations shall be supervised by the Head of the Mathematics Department of the institution at which they are held, or by some one appointed by him to do so.

Sec. 4. The examinations shall be made out by persons chosen by the Executive Council and their names are to be made known to the members of the Association.

Sec. 5. The time limit of each examination shall be set by the author of the examination.

Article XIII. Grading

Sec. 1. The preliminary examinations of each region shall be graded by the three elected members of the Executive Council not of that region.

Sec. 2. Each of the final examinations shall be graded by the four elected members of the Executive Council and the final grade for each paper shall be the average of these grades.

Sec. 3. The rules regulating the grading of the papers shall be made by the members of the Executive Council.

Article XIV. Eligibility of Students

Sec. 1. Students competing in algebra and trigonometry must be below the third year in mathematics.

Sec. 2. The contests shall be limited to undergraduate students of the members of the Association.

Sec. 3. A student who participates in any one of these examinations cannot participate in any other except the comprehensive examination and the final contest.

Sec. 4. The number of contestants in the preliminary examinations shall not exceed two students for each examination.

Article XV. Prizes

Sec. 1. The winner of the final contest shall be presented a cup for temporary possession. The Executive Council shall have authority to decide the winner in case of a tie. To the school that is first in winning the cup for three times, the Association shall present it for permanent possession. The same process will then be repeated with the second cup, etc.

Sec. 2. The members of the Association shall recognize the students taking part in these contests by awarding to each of them a "letter", or an equivalent insignia.

Article XVI. Aim

Sec. 1. It shall be the aim of each member of the Association to conduct its work in a most ethical and scholarly manner, bending its efforts always towards true scholarship, sincerity of purpose, and rational and balanced idealism.

Article XVII. Amendments

Sec. 1. This constitution may be amended at any annual meeting by two-thirds of the votes cast by the members present.

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BOOK REVIEW DEPARTMENT

Edited by
P. K. SMITH

College Algebra, by Raymond W. Brink.

This book is an excellent text for the student who has had either two or three semesters of algebra in high school whether he intends to pursue the study of mathematics beyond the more elementary courses or not.

The first portion of the book reviews the material of high school advanced algebra but being written from a more mature point of view also serves as an introduction to college algebra.

The terminology is useful to the student who proposes to continue the study of mathematics. He becomes familiar with terms so often discovered for the first time in advanced courses.

Care has been used to present complete proofs or, where the complete discussion is beyond the scope of the book, to state explicitly the assumptions made and the consequent limitations of the theorem. In the few instances where the proof is too advanced for the course, attention has been directed to the omission and a reference made to where the proof can be found.

The subjects of the various chapters and the arrangement of the material therein permits a wide variety of courses. For example, later chapters are independent of one another, paragraphs not essential to the continuity are starred, problems are graded with the easier ones in each group first, and they occur in pairs with the answers to the odd numbered ones only.

The chapter on complex numbers is particularly good. It includes the polar representation of a vector, the product, quotient, powers and roots of vectors with a number of geometric figures.

As a matter of fact, graphs are used extensively in this book but with the idea of interpreting, checking, and guiding rather than as a means of obtainng results.

An adequate treatment of infinite series, partial fractions, and determinants for use in calculus and differential equations is given in the chapters devoted to these subjects, although there is no discussion of the matrix.

The functional relation is emphasized by continually using the functional language and notation.

In addition to the usual subjects there is a chapter on the mathematics of investment and also one on probability.

The book contains 445 pages, is 5"x 8", and dark red in color. Italics and heavy type are used for emphasis and headings.

—R. H. Knox, Jr.

PROBLEM DEPARTMENT

Edited by
T. A. BICKERSTAFF

This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and solve problems here proposed.

Problems and solutions will be credited to their authors.

Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

Problems for Solution

No. 43. Proposed by Professor C. D. Smith, Mississippi State College.

Given a quadrilateral, $P_1P_2P_3P_4$ with the usual rectangular co-ordinates.

Find the conditions under which,

$$x_1x_3 - x_2x_4 + y_1y_3 - y_2y_4 = 0$$

No. 44. Proposed by T. A. Bickerstaff.

A, B, and C are stationed at the three vertices of an oblique triangle. Starting simultaneously, A moves toward B, B moves toward C, and C moves toward A; all moving at the same rate. Investigate their paths.

No. 45. Proposed by C. D. Smith, Mississippi State College.

Show that the Brocard points of a triangle may be taken as special cases of Miquel points.

No. 46. Proposed by C. D. Smith:

Given a square ABCD of side a and quadrilateral ABEF such that A and B are vertices of the square, E is on CD and F is on AD. Required, a formula for bisecting the quadrilateral by a line GG′ parallel to AB, and discuss of the special cases where

$$FA \begin{matrix} > \\ = \\ < \end{matrix} GV$$

He Studied Mathematics
---then law

Now internationally known in the legal world, asked if he ever used his knowledge of mathematics, he has given this answer:
"Its technical formulæ, very rarely; but its logic, method, and way of reasoning, almost every day ... Mathematics as a tool for training can yield to no other branch of knowledge handled in the curriculum of an educational institution."

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